

# Limitations of Model Predictive Controllers

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## Introduction

Multivariable Model Predictive Controllers (MPC) have been used to control process plants for several years now, and are currently available from several software vendors. Many of these vendors imply that the MPC applications have been an unqualified success, and that model predictive controllers are the preferred solution for all control problems.

While model predictive controllers can be effective in the proper situation, they have limitations that are generally not mentioned by the vendors marketing these controllers. Difficulties with operation, high maintenance cost, and lack of flexibility can result in fragile controllers that are not profitable. Discussed in this article are some of the limitations of model predictive controllers – not just in the algorithm itself, but also in the justification of these controllers, applicability on chemical process systems, and subsequent maintenance. Also discussed are technologies for alleviating some of these limitations.

## Controller Justification

Many vendors claim significant financial benefits from MPC applications - usually in the range of one million dollars per year per application. However, details of the financial benefits are usually scant, and the following questions are often overlooked:

1. Was a multivariable model predictive controller required? A multivariable controller is *only* required if the process is coupled; but significant coupling is not a guaranteed attribute, as an objective of most plant designs is to *minimize* interaction. There are standard interaction measures, such as the RGA<sup>1</sup>, which indicate interaction. These techniques do not require any additional information (just the plant steady state gains, which are required for the MPC controller), and are simple to calculate and interpret. Certainly a standard part of the justification of any multivariable controller should be an investigation into the coupling of the plant.
2. What is the quality of control, both before and after installation of a MPC controller? This is surprisingly difficult to measure, as it depends to a very large extent on the level of disturbances and setpoint changes. Usually output error variances are quoted to indicate the improvement in control, but these numbers are almost meaningless (worse, they can be manipulated to draw any conclusion) as they depend mainly on the disturbances/setpoint spectrums at the time the data was taken. A much better measure of controller performance is the Harris Controller Performance Index<sup>2</sup>, which compares the performance of the controller to a theoretical best performing (i.e., minimum variance) controller, and is invariant to the level of disturbances or setpoint changes.

3. Were any improvements made to the regulatory layer prior to installing the MPC? Effort is often made to tune up the lower regulatory layer, repair sticking valves, etc. before the MPC controller is applied. The improvement attributed to the MPC may therefore be largely the result of the simple engineering of ensuring the basic controls function properly.
4. Were any measurements added? Benefits are commonly claimed for a MPC controller pushing the plant to a new measured constraint, but is this benefit due to the MPC controller, or is it due to new knowledge of the constraint?
5. Does constraint pushing require a linear program (LP)? Often the optimal operating point of a process is invariant and can be determined with a minimum of process knowledge. Construction and commissioning of an on-line LP is excessive if the optimum point is simply keeping the feed valve wide open. For more complex uncoupled systems, override controllers (standard in DCS systems) are excellent for pushing a unit to its limits.
6. Are the benefits due to improved control, or can they plausibly be attributed to different feedstocks, etc. A 1% improvement in conversion or throughput can have significant dollar benefits, but this number may be suspect if the accuracy of the flowmeters is the normal 3% to 5%, and the feed quality varies in some unknown way.

MPCs are often applied regardless of new information, or reconsideration of old information. Such need not be the case. If information from plant tests or additional measurements indicate a simpler way to control the process is available, then this path should be followed. Effort in improving the regulatory layer is almost always a worthwhile expenditure in itself. Similarly, dynamic models obtained from plant tests often lead to additional insights and can be used to tune existing controllers.

### **Installation and Maintenance Expense**

Application of MPC controllers generally requires weeks or months of plant tests, controller building, and post-installation tuning before they are suitable for operations. This is, of course, on top of the investment in DCS systems and perhaps higher level computers to support these applications.

Now consider what is involved in maintaining the controller. In particular, the control engineer wishes to know whether the controller is excessively detuned, whether model mismatch is affecting the performance, or whether the plant is at its true optimum. As mentioned in the Controller Justification section, just measuring the controller performance can be challenging. The Harris Controller Performance Index previously noted is strictly speaking only applicable for the SISO (Single Input Single Output) case; extensions to multivariable control have been proposed<sup>3</sup>, but they are difficult to apply, and some of the underlying assumptions of these techniques are not realized in practice. Tools for determining which of the models (of the tens or hundreds) may be in error are primitive or non-existent, and a common procedure is to simply retest the entire plant (at significant cost) whenever the performance falters too much.

Determining whether the controller is driving the plant to the true optimum can also be difficult, as the LP always determines the *model* optimal solution, if there is one. The problem then reduces to determining whether the LP model is correct, but this is generally difficult to determine unless new plant tests are taken.

Commonly, the first step to analyzing controller performance is to construct or buy a database that logs all controller variables at all times. Simple enough, but it results in a huge amount of data – one estimate is that the typical application will generate 1 Gigabyte of data per month<sup>4</sup>. Clearly, this is far too much for an engineer to analyze, and computer routines need to be employed. Unfortunately, less detail is forthcoming as to what applications and algorithms are intended for this data. Furthermore, little or no information may be contained in this happenstance data<sup>5</sup>. Controller feedback, unknown disturbances, and model mismatch all combine in a Jackson Pollack style of data presentation that defies separation or categorization.

None of these issues are fatal shortcomings in themselves. If the financial incentive is there, the necessary resources should be supplied to install and maintain the controller. But they do present a bar that must be (continually) surmounted – it is not enough to consider the one-time expense of controller installation in evaluating the economics, but rather the life cycle cost as to whether the controller maintains a sufficient return on investment.

### **Controller Structure Limitations**

On the surface, MPC controllers appear complete: optimality in both economic and regulatory control, ability to handle all plant inputs and outputs, and a centralized structure for coordination. But there is no guarantee that the (somewhat arbitrary) mathematical optimizations of MPC translate well into engineering objectives, nor that the rigid structure of MPC controllers matches the engineering design problem at hand.

Ricker<sup>6</sup>, in an unbiased comparison of MPC to conventional Proportional Integral (PI) control, found “there appears to be little, if any, advantage to the use of non-linear model predictive control (NMPC) in this application [the Tennessee Eastman design problem]. In particular, the decentralized strategy [PI control] does a better job of handling constraints – an area in which NMPC is reputed to excel.” No single set of MPC controller weights and constraints could provide the desired performance for the many competing goals and special cases of the plant. This view was also held by Lyuben et al. in their recent text<sup>7</sup>, where they found that a wide variety of plants could be adequately controlled by the intelligent application of single loop controllers.

MPC has performance problems in even relatively simple situations. Consider, for instance, three common control situations that are handled well by conventional or Application Specific Controllers (ASC) but are handled poorly by MPC control:

## 1. Cascade Control Loops

An example of the ubiquitous cascade control loop is shown in Figure 1: a composition controller outputs to a temperature controller, which in turn outputs to a flow controller. As the analyzer contains a significantly longer deadtime than the temperature controller, the temperature controller is much more effective at rejecting disturbances. The analyzer controller effectively bias-updates the temperature controller. Furthermore, there is inherent robustness in the design; when the analyzer fails, the temperature controller can continue rejecting much of the disturbances.

Now consider when this cascade loop is put into a MPC framework. Generally both the analyzer and temperature are controlled, but only the analyzer has tight bounds, as it is not possible to control both analyzer and temperature to a setpoint. However, deadtime compensation is now lost as the temperature remains in bound during small disturbances – the controller will not respond until the analyzer measurement changes. Larger disturbances result in both the analyzer and the temperature at a constraint, and now the controller must attempt to control both measurements to specified values. Clearly this is not possible, and the controller will generally move excessively attempting to achieve the incompatible objectives.

Finally, consider what happens when the analyzer fails. The MPC controller will move the temperature to an upper or lower bound (depending on LP values). This is clearly undesirable, so logic must be added to clamp the upper and lower temperature bounds to the current value whenever the analyzer fails. Complicated enough, but should the logic be reversed when the analyzer becomes good again? Are operators aware of these complicated failure/recovery modes?

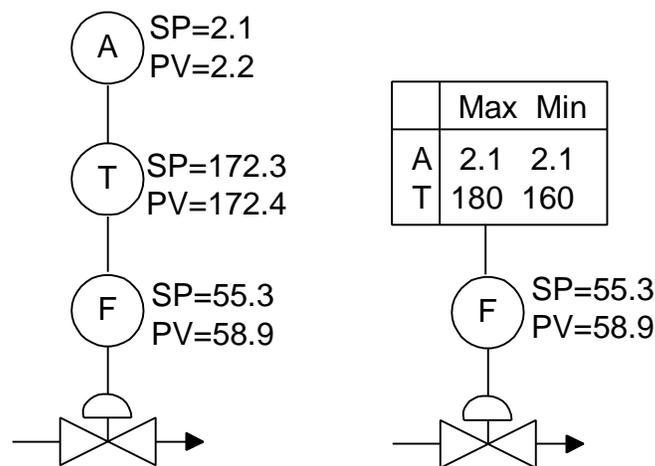


Figure 1: Cascade and MPC control loops.

## 2. Surge Vessel Level Control

A natural application of MPCs would appear to be for surge vessel level control (Figure 2). Here the objective is to keep the level between bounds (which are handled by the LP), while dampening the rate of change of the outlet flow (which is handled by the dynamic regulatory controller). Unfortunately, MPCs do not function this way. The LP will drive the process to one of the bounds at steady state, and as the desired steady state is generally 50%, the LP bounds must both be set at this value.

Effectively, the MPC controller in this case is a PI controller, and one with a large amount of integral action (as is the nature of MPC controllers). What is required is a PI controller with a large amount of proportional action. A proper surge vessel level controller would be an even better solution. These non-linear controllers solve a mathematical optimization that maintains the level between bounds while minimizing outlet flow changes, and are small enough to run in DCS systems.

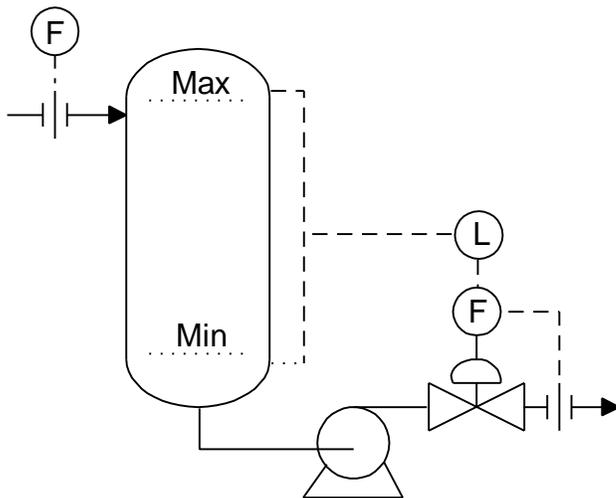


Figure 2: Typical Surge Vessel Level Control Configuration

## 3. Disturbance Rejection

Given the touted benefits of MPC, it is somewhat surprising to find that they can be quite sluggish in rejecting long drifting disturbances. This is not due to model mismatch or excessive detuning, but is an artifact of the controller formulation. A key assumption of commercial model predictive controllers is that all disturbances are step disturbances, resulting in the prediction of the future disturbance being equal to the current calculated disturbance. Clearly this is wrong for long drifting disturbances, resulting in the controller underestimating the size of the disturbance, and thus underestimating the required response.

The algorithms proposed by researchers to overcome this limitation generally replace the step disturbance model with a low order transfer function<sup>8</sup>. In theory, this will result in tight control, but it may be very sensitive to model mismatch and noise. It suffers the additional weakness of requiring knowledge of the disturbance response, but this may be difficult to obtain as many disturbance sources are unmeasured and therefore unmodeled (if measured, they will likely already be incorporated as feedforward variables).

Another difficulty with MPC controllers is the large number of tuning parameters required. Ironically, one of the initial promises of MPC is that they would be simple to tune, but current controllers contain a multitude of tuning parameters to determine their behavior. A partial list of the parameters is as follows:

Input Parameters	Output Parameters
Move Suppression	Output Weight
Input Upper Bound	Output Upper Bound
Input Lower Bound	Output Lower Bound
Maximum Step Size	Equal Concern Error
LP Costs	Funnel Shape or Constraint Zones

Like regulations in a bureaucracy, each parameter may make sense in isolation; together they present a considerable challenge to any control engineer. All these parameters interact, and the magnitudes of many of the parameters have meaning only in relation to other parameters. And there can be hundreds of parameters, as each output and input has a complete set.

### Process Limitations

It is always tempting to include all plant inputs and all relevant plant outputs in a MPC controller, and then tune the controller to achieve desired goals. Lurking underneath this strategy lies a critical flaw that has disabled many controllers while leaving only the barest traces of its presence. This *dues ex machina* is ill-conditioning, a familiar quality to students of linear algebra, but less widely known to control engineers.

Basically, the conditioning of a matrix (or a system) represents its sensitivity to model mismatch, particularly in inverting the matrix. Since model predictive controllers are essentially psuedo-inverses of the plant model, this matrix measure has deep applicability in controller design and application.

The numerical measure of ill-conditioning, known as the condition number, is the ratio of the largest singular value of a matrix to the smallest. Well-conditioned matrices have a condition number that approaches one; the higher the number the more ill-conditioned is the matrix. Unfortunately, there is a shortcoming to this measure – it is heavily dependent on the scaling of the matrix. That means you can get different condition numbers merely by rescaling the inputs or outputs. There are strategies to minimize the

effect of scaling, but no absolute method. At any rate, the condition number can provide a guide for ill-conditioning, but there is no set number separating a well-conditioned matrix from an ill-conditioned one.

Two examples will help to illustrate this phenomenon. The first is a poorly conditioned 2x2 matrix (condition number = 199), as shown in Figure 3. When a 1% perturbation is added to this matrix, the inverse changes by approximately 100%.

$$A = \begin{bmatrix} 1.0 & 0.99 \\ 0.99 & 1.0 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 50.25 & -49.75 \\ -49.75 & 50.25 \end{bmatrix}$$

$$A + 0.01 * I = \begin{bmatrix} 1.01 & 0.99 \\ 0.99 & 1.01 \end{bmatrix} \quad (A + 0.01 * I)^{-1} = \begin{bmatrix} 25.25 & -24.75 \\ -24.75 & 25.25 \end{bmatrix}$$

Figure 3. The matrix A is only slightly perturbed, but the inverse is significantly different.

The second example is for an actual process, as shown in Figure 4. Here the objective is to control two temperatures of a distillation column using 2 inputs, reflux flow and reboiler duty. While there are two separate inputs, both have approximately the same effect on the column. That is, they either heat it up or cool it down, with the reflux gains being approximately the negative of the reboiler gains. Mathematically, the matrix for this process could be inverted, but small changes would lead to large differences in the inverted matrix, and therefore in the controller action. This characteristic is true of all distillation columns (particularly high purity ones), and is one reason why distillation columns often have only one temperature or composition controller.

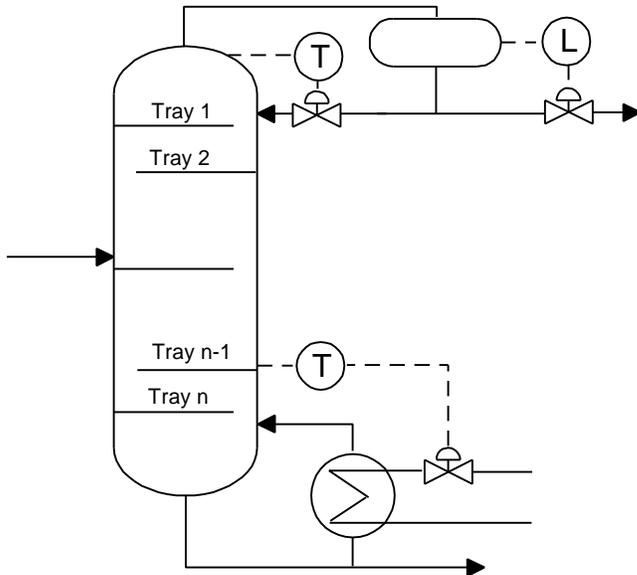


Figure 4: Independent control of both temperatures is difficult as both the reflux and reboiler either heat up or cool down the entire column.

Unfortunately, this aspect of control is extremely common – tightly coupled processes are usually ill-conditioned to some extent. So the control engineer who requires MPC control to break the interactions need also check for ill-conditioning. There are commercial controllers that are said to be more robust as they check for ill-conditioning online<sup>9</sup>, but there are two difficulties in their usage:

1. The control engineer must specify the condition number as a (yet another) tuning parameter. There is little guidance to what this number should be, as it depends on the scaling of the problem etc. It may even need to be changed on-line as various inputs and outputs are brought into and out of the controller.
2. It is difficult or impossible to predict the action of the controller. At some point, the controller will determine that part of the plant is ill-conditioned, and not control that part of it. What part this is, and at which point it will be uncontrolled is very difficult to determine, particularly by the operations staff, whose job it is to control the plant.

A better technique is to check conditioning in the design stage. All that is required is the plant steady-state gains, and a moderate amount of standard linear algebra. The results from this mathematical analysis can be combined with the control engineers process understanding of the process to yield a controller that never is called upon to control an ill-conditioned system.

The LP portion of a MPC controller can also exhibit characteristics similar to ill-conditioning. This occurs when the objective function is closely aligned with one of the constraints (Figure 5). In this situation, small changes of the plant result in the optimum being at different vertices of the constraint set, and a very different set of setpoints sent down to the regulatory portion of the MPC. Again, there are techniques for determining this potential condition in the design stage<sup>10</sup>.

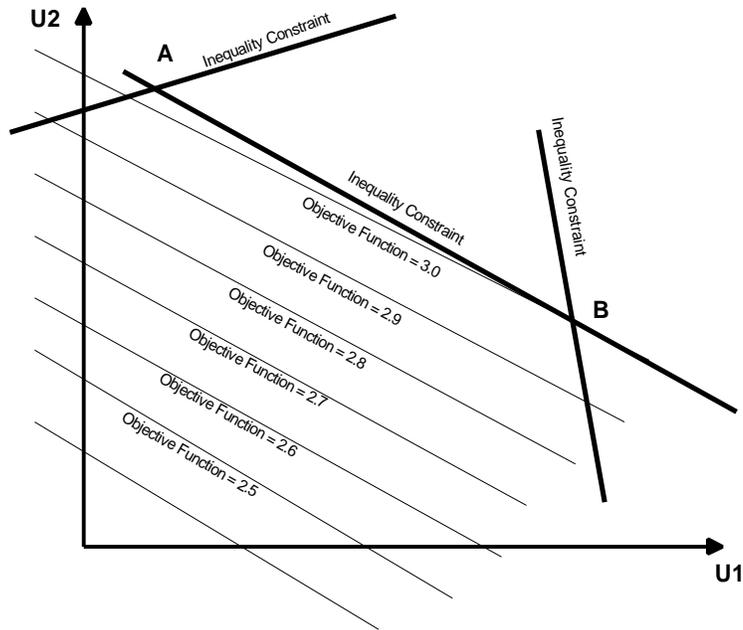


Figure 5: A slight change in the objective function or constraints can move the optimum point from Point A to Point B, as the objective function is in rough alignment with a constraint.

### Operator Interface

The promise of MPC controllers is that they would relieve the operators of many of their functions – indeed there were worries at one point that the operator skill set would ossify as more and more of the plant control would be performed by the MPC controller. These fears proved to be unfounded as MPCs often requires more operator intervention and have added to the operator workload. There are two reasons for this:

1. The unpredictability of the model predictive controller, especially coupled with an LP, requires the operator to ensure that the controller is behaving as desired. In fact, the Abnormal Situation Management Consortium has found that MPC controllers can contribute to escalation of upsets<sup>11</sup>.
2. The controller can behave poorly in the face of model mismatch and large plant disturbances. This is the flip side of high performance – like any highly specialized species, the mortality rate is high when the environment is different than the design case.

One of the advantages of single loop control is its transparency – if the level is too high, the valve opens. Once these same loops are put into a multivariable controller, this transparency vanishes – if the level is too high, the valve may open, or the perhaps the reboiler duty may increase, or the feed may cut back, or perhaps some combination of the above. Indeed, if the only action of the controller is to open the valve when the level is too high, then there is really no point in putting that loop into the MPC. If the controller always works flawlessly all the time, then operators will accept it. But it only needs to fail one times in 20

for operators to develop a distrust of the controller, and to either watch it more closely, or turn it off whenever a disturbance enters the process.

This common de-facto standard procedure of turning off a controller whenever moderate or large disturbances enters the plant nullifies much of the perceived advantage of better disturbance rejection. Quoted numbers of 90-95% on-line time should be interpreted in the light of the fact that most processes have very little disturbances 90% of the time, and require little control during these periods.

### **Example: Fluidized Cat Cracker Control**

Fluidized Cat Crackers are common applications for MPC control as they contain several inputs and outputs and are often the dominant economic unit in a plant. Usually not undertaken in these applications is an investigation into the coupling and conditioning of the system – as stated before, an MPC can only show benefits if the system is multivariable *and* well conditioned.

A typical process gain matrix, taken from the MPC plant tests for an industrial cat cracker, is shown as Figure 6a. The accompanying RGA matrix indicates moderate coupling (a value close to one in each row or column indicates the corresponding input/output can be paired in single loop controllers). While the RGA indicates a MPC would likely result in noticeable improvement in reducing interaction, the condition number is high (220), indicating that the controller would also be sensitive to model mismatch.

This poor conditioning is mainly due to the CO output and the regenerator temperature output having similar responses. One way to remove this ill-conditioning is to put the Regenerator Temperature into the regulatory layer, transforming it from a dependent (or controlled) variable, to an independent (or manipulated) one. The regenerator temperature controller in turn cascades down to the delta pressure controller. This has favorable dynamics as the dynamics of the regenerator temperature controller are typically much faster than the dynamics of the remaining dependents.

The gain matrix for this system is shown as Figure 7a. Now the system has 4 inputs and 3 outputs, or one extra degree of freedom. In general, feed is a poor choice for regulatory control of cat crackers, as it has very long dynamics and affects both upstream and downstream units significantly. It therefore is usually used as an optimization variable or to alleviate a constraint somewhere in the plant.

The RGA for the system with the feed removed is shown in Figure 7b. There is a value close to one in each row or column, indicating that a decoupled set of pairings would be CO/Regenerator Temperature, Reactor Temperature/Fuel Gas Flow, and Cyclone Velocity/Fractionator Pressure. The condition number is also quite low (11.9), indicating that this new system is robust to model error. This single-loop

configuration is quite common for these units – the mathematical analysis merely confirms what control engineers had found through experience.

In summary, the single-loop configuration has the following advantages over a MPC controller:

1. The performance of the single loop controllers would be almost the same as a multivariable MPC controller as the system is largely decoupled. Indeed, the single loops will likely perform better, as the Regenerator Temperature control would be much more effective at rejecting disturbances.
2. The single loop structure is much more robust, as indicated by the large reduction in condition number between the two structures (from 220 to 11.9).
3. Single loops are vastly easier to construct and maintain. And they are easier to operate, as each loop has a single objective, and a single handle to achieve that objective.

Process Gains	Feed to Unit	Fuel Gas Flow	Rxn/Rgn ? P	Fractionator Pressure
CO In Flue Gas	-0.0119	-0.0159	0.1559	-0.2704
Reactor Temp	-0.2725	0.01837	-0.00387	-0.1777
Cyclone Velocity	-0.0526	-0.0119	0.0725	-0.7244
Regenerator Temp	-0.1589	0.01986	-0.17871	0.5699

Figure 6a: Process Gains for original configuration.

RGA Matrix	Feed to Unit	Fuel Gas Flow	Rxn/Rgn ? P	Fractionator Pressure
CO In Flue Gas	0.06	0.88	0.49	-0.43
Reactor Temp	0.02	0.93	-0.02	0.07
Cyclone Velocity	0.12	0.54	-0.49	0.84
Regenerator Temp	0.81	-1.34	1.02	0.52

Figure 6b: RGA Matrix for the original configuration. Moderate coupling is indicated by most rows or columns not having elements close to unity. Condition number for this system = 220.

Process Gains	Feed to Unit	Fuel Gas Flow	Regenerator Temperature	Fractionator Pressure
CO In Flue Gas	0.1505	0.01425	-0.8727	0.2268
Reactor Temp	-0.269	0.180	0.2165	0.1900
Cyclone Velocity	-0.1167	-0.0384	-0.0406	0.9556

Figure 7a: Process Gains when the Regenerator Temperature is a manipulated variable.

RGA Matrix	Fuel Gas Flow	Regenerator Temperature	Fractionator Pressure
CO In Flue Gas	0.0277	1.073	-0.10
Reactor Temp	0.9157	0.0331	0.0511
Cyclone Velocity	0.0566	-0.106	1.049

Figure 7b: RGA Matrix for above process. The RGA here indicates a decoupled process, as there is a number close to unity in each row and column. The condition number for this system is 11.9.

### Example: Binary Distillation Column

Consider again the binary distillation column shown in Figure 4. A typical process model, taken from a study by Skogestad and Morari<sup>12</sup>, give the following process model and Relative Gain Array:

Process Gains	Reflux	Reboiler Duty
Overhead Comp	0.8754	-0.8618
Bottoms Comp	1.0846	-1.0982

Figure 8a: Process Gains for the standard LV configuration controlling compositions.

RGA Matrix	Reflux	Reboiler Duty
Overhead Comp	35.5	-34.5
Bottoms Comp	-34.5	35.5

Figure 8b: RGA Matrix for above process. The high values indicate a very coupled, ill-conditioned process (Condition Number = 160.5).

Essentially, no controller can effectively control the above 2x2 system. It is however possible to obtain a controllable system by modifying the control structure. Rather than manipulating the reflux flow, Wolff and Skogestad<sup>13</sup> show that implementing a secondary temperature controller between tray 8 temperature and the reflux flow gives the decoupled, well conditioned system shown below:

Process Gains	Tray 8 Temp	Reboiler Duty
Overhead Comp	-0.1021	0.00728
Bottoms Comp	-0.1267	-0.513

Figure 9a: Process Gains for the modified configuration controlling compositions.

RGA Matrix	Tray 8 Temp	Reboiler Duty
Overhead Comp	0.983	0.017
Bottoms Comp	0.017	0.983

Figure 9b: RGA Matrix for modified process, indicating that the system is decoupled (Condition Number = 6.25).

In addition to better decoupling and conditioning, Wolff and Skogestad also show that the modified system has much better disturbance rejection capabilities. Further improvements in the control may be achieved by incorporating various ratios (i.e., reflux ratio, reboiler duty to bottoms flow) into the control scheme. And as mentioned previously, the cascade control structure also has the benefit of being much more resilient to analyzer failures.

All these modifications may be undertaken with standard DCS control algorithms, and are simple to operate. If it is desired to add an MPC to this system, then the control engineer must decide whether to use the original control structure (which is too ill-conditioned to be effective) or the modified one (which does not require a multivariable controller).

## Conclusions

The black-box formulation of Model Predictive Controllers, often coupled with simplistic and inefficient model identification routines<sup>14</sup>, inhibit many of the design decisions once made by control engineers. Knowledge of the process, analysis of signals and responses, and construction of Application Specific Controllers often are not given sufficient consideration in the construction of these fixed structure controllers. Furthermore, the common assumption that employing only a single controller technology will simplify maintenance may be erroneous if the design is wrong for the application, and there are no maintenance tools.

This would be acceptable if Model Predictive Controllers always worked, but the evidence is that they require significant resources to install and maintain, can be difficult to operate, and often display poor performance. Of course, the shortcoming is not in applying these über-controllers – they can be extremely profitable and effective in the right circumstances. The shortcoming is rather in applying *only* these controllers, regardless of the needs of the problem, and regardless of the inherent tradeoffs.

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